# Project Summary

## Diagrammatic and geometric techniques in representation theory

Ben Webster
University of Virginia

The PI plans to research the representation theory of deformation quantizations of symplectic singularities. The long term research objective of this program is proving the conjecture of the PI and his collaborators that there is a duality operation on symplectic singularities. At moment, the most important property of this proposed duality is its effect on the representation theory of associated algebras: "categories $\mathcal{O}$" attached to the singularities should be Koszul dual (that is, possess a very special equivalence of categories). Many special cases of this duality are well-understood, but what links them remains to be investigated. This duality also has an explanation in terms of field theories in physics, where the dual varieties appear as the Higgs and Coulomb branches of the moduli space of vacua for $\mathcal{N} = 4$ supersymmetric 3 dimensional field theories.

This research is also tied up in the exploration of individual examples of these singularities; in these our proposal would be a "geometrification" and "categorification" of well-known dualities in mathematics, such as Schur-Weyl duality, rank-level duality and Gale duality. Our perspective also provides a fruitful approach to topics as diverse as the representation theory of symplectic reflection algebras and the Rouquier-Khovanov-Lauda categorification of quantum groups, and has applications as far afield as low-dimensional topology, since it allows the construction of vector-space valued knot invariants which categorify quantum knot invariants, such as the colored Jones polynomial.

**Intellectual merit:** Symplectic algebraic varieties arise naturally from many different areas of mathematics. Quiver varieties provide a geometric context for actions of Lie algebras, leading to canonical bases and to actions on categories of sheaves, while hypertoric varieties have natural ties to the combinatorics of hyperplane arrangements. Our constructions play a key role in these picture, providing new geometric insight to such phenomena as Gale duality in combinatorics, level-rank duality in representation theory, and homological invariants of knots and links. Furthermore, they connect to exciting work in mathematical physics by Witten, Gaiotto, Dimofte and others. The PI's interests span a range of subfields from low dimensional topology to geometric representation theory and the proposed research will create unexpected connections between these fields.

**Broader impacts**: The PI has been active in sharing mathematics through the internet (he is a board member of `mathoverflow.net`, and through expository lectures from the high school to graduate level, as well as his usual duties teaching and advising students. He has served as a mentor for UVA's Mentoring Institute for graduate students in underrepresented groups. He has organized several conferences in recent years and is also involved in graduate education at UVA as a teacher and member of the graduate committee.